Short Papers

A Higher Order Approximation for Waveguide Circulators

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Abstract—**Earlier work on the analysis and experimental performance of 3-port and 4-port waveguide circulators over the waveguide bandwidth has shown some discrepancies. The computed performance tended to be too optimistic. It is shown that an extension of Davies' analysis to include higher order cylindrical modes and evanescent modes in the waveguide arms yields excellent correlation between theory and practice.**

I. Introduction

Extending the work of Auld [1] and Davies [2], the behavior of radially inhomogeneous waveguide circulators has recently been reported [3]. The ferrite/dielectric/metal structures uniformly extend the full height of the waveguide (Fig. 1) and the rectangular waveguide excitation is assumed to be the TE_{0p}, i.e., no field variation in the direction of the static magnetic field in the ferrite. This assumption permits an analysis in terms of cylindrical modes in the ferrite rod that are TM_z. The results of this investigation were very encouraging using azimuthal variations of order $n=0, \pm 1$, but there were two noticeable types of discrepancy. The first was the occasional appearance of unpredicted spikes in the measured performance; the second was an unpredicted hump in the curve of measured reflection coefficient or isolation, with an associated increase in insertion loss. The spikes, which are well known to experimental designers, have since been identified as resonances of the ferrite post open resonator (FPOR) hybrid modes [4]. They are trapped modes and cannot participate in circulator action. They may be excited by an axial nonuniformity, and perhaps the most frequent cause is imperfect contact between the ferrite rod and the adjacent conducting surfaces. The "hump" discrepancy could not be explained using the FPOR modes, and it will be shown below that by including higher order circulating modes, i.e., cylindrical modes with $|n|>1$ and zero z variation, and the first two evanescent waveguide modes (TE_{0p}), the overall agreement is improved significantly.

II. Analysis

The general technique consists of solving the boundary value problem inside the geometrically symmetrical junction, which then enables the junction eigenvalues to be calculated [2]. From the eigenvalues, the elements of the scattering matrix of the m-port junction can be determined using the results in the appendix of Auld's paper [1].

The fields within the junction region, i.e., within the central dotted region of Fig. 1, are described by an infinite set of cylindrical modes. The fields in each waveguide near the junction are also described by an infinite set. If the m-port junction is excited with a particular eigenvector of the scattering matrix, the electric field of the eigensolution differs only by a phase factor for each port. Because of this fact, and using superposition of the eigensolutions, it is sufficient to discuss the fields in one waveguide together with those in the cylindrical region between it and the junction axis. Consequently, the problem may be solved by matching each cylindrical mode exactly to the complete set of rectangular waveguide modes. An infinity of equations results (one for each cylindrical mode) each involving an infinity of unknowns (the amplitudes of the waveguide modes):

Manuscript received December 29, 1971.
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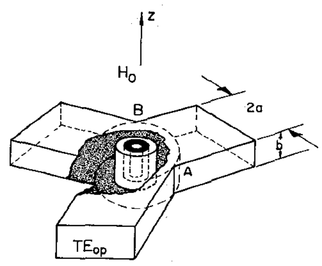

Fig. 1. Symmetrical waveguide junction with radially inhomogeneous composite ferrite post. The central rod may be a metal pin or a dielectric rod. The shaded inner annulus is a ferrite tube which is surrounded by a dielectric sleeve.

$$\sum_{p=-1}^{\infty} A_p(q)G_p = 0, \qquad q = 0, \pm 1, \pm 2 \cdots. \tag{1}$$

A_p and G_p are given in [2]. The calculation of the eigenvalues requires the ratio of the reflected and incident dominant waveguide modes, $p=-1$ and $p=1$. Using Cramer's Rule, the $(r+1)$th iterate for the successive approximate solution is given by Davies' equation (29), and Davies' first approximation took the form $\lambda_j = -A_1(q)/A_1^*(q)$ where q was not fixed, but chosen to minimize the expression $n=|mq+j|$. That is, he considered only the lowest possible order of cylindrical modes. In an attempt to improve the agreement between predictions and experiment, especially as related to the "hump" discrepancy, the next higher approximation was taken.

Successive approximate solutions to the infinite system can be derived from one of the linear equations, from three equations, and so on, the number of equations required always being odd. Thus the second approximate solution takes the form

$$\frac{G_{-1}}{G} = -\frac{\begin{vmatrix} A_1(1) & A_2(1) & A_3(1) \\ A_1(0) & A_2(0) & A_3(0) \\ A_1(-1) & A_2(-1) & A_3(-1) \end{vmatrix}}{\begin{vmatrix} A_{-1}(1) & A_2(1) & A_3(1) \\ A_{-1}(0) & A_2(0) & A_3(0) \\ A_{-1}(-1) & A_2(-1) & A_3(-1) \end{vmatrix}}$$

where q must have the values 0, ± 1 in order to obtain three equations. The inclusion of the terms $A_2(q)$ and $A_3(q)$ means physically that the fields of the first two evanescent waveguide modes ($p=2, 3$) are now included in the boundary value problem.

This extension differs slightly from Davies' approach in that he intuitively let q vary to obtain the lowest order mode associated with each eigensolution. We incremented q through the appropriate range of values without minimizing $|mq+j|$. Thus a comparison of the first and second approximations in terms of the number of waveguide and cylindrical modes included in the calculation yields: for the first approximation (FA), the $n=0, \pm 1$ for a 3-port and $n=0, \pm 1, \pm 2$, for a 4-port; for the second approximation (SA), the incident and reflected dominant waveguide modes, the first two evanescent waveguide modes, $n=0, \pm 1, \pm 2, \pm 3, 4, 5$ for a 3-port and $n=0, \pm 1, \pm 2, \pm 3, \pm 4, 5, 6, 7$ for a 4-port.

III. Experimental Results

The higher approximation was used to compute the performance of some 3-port devices for which the experimental behavior and the FA predictions were already available [3]. There was an overall improvement in the prediction, and the correlation with experiment was generally excellent. TT1-390 ferrite was used in the experiments, and the nominal parameters of this MgMn material are: $M_s = 1.71$

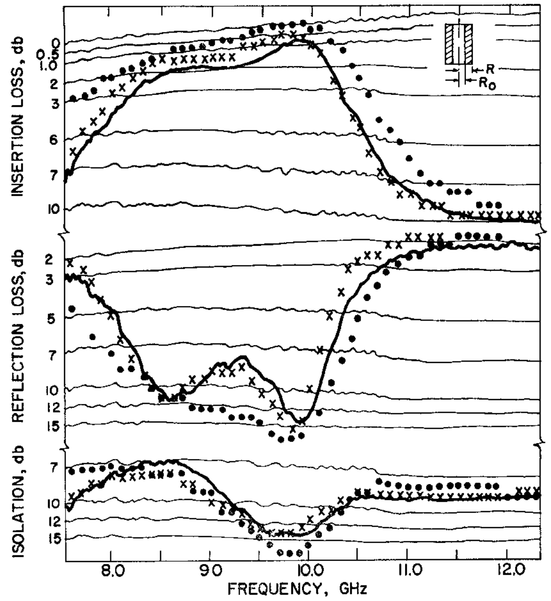

Fig. 2. 3-port performance of a hollow TT1-390 ferrite tube. $R=0.350$ cm, $R_0=0.150$ cm, $\epsilon_{r0}=1.0$, $H_0=2\times10^5$ A/m. (———, measured; · · · , first approximation; × × ×, second approximation.)

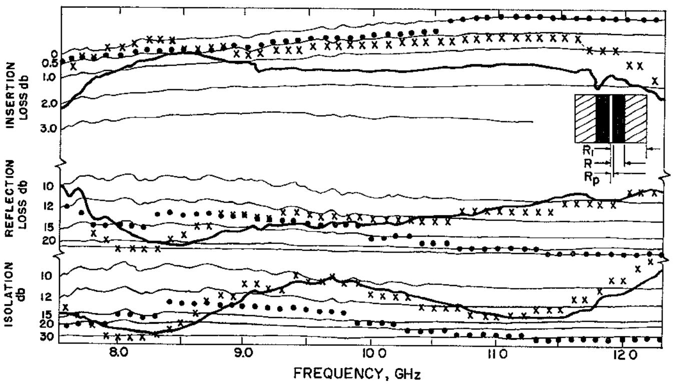

Fig. 3. 3-port performance of TT1-390 ferrite rod, a metal pin, and dielectric sleeve. $R_1=0.775$ cm, $R=0.275$ cm, $R_p=0.025$ cm, $\epsilon_{r1}=2.08$. (———, measured; · · ·, first approximation; × × ×, second approximation.)

$\times10^5$ A/m, $H=5.4\times10^5$ A/m, $\epsilon_r=12.7$. However, upon request, the manufacturer supplied the following more accurate value of saturation magnetization. $M_s=1.76\times10^5$ A/m. In the figures, the closed circles mark the FA, the crosses mark the SA, and the continuous lines show the measured performance.

The main feature of the improvement in Fig. 2 for the hollow TT1-390 ferrite tube is that the "hump" discrepancy in the reflection loss curve is resolved. There is also much better agreement at both ends of the frequency range. The thin-pin structure of the type shown in Fig. 3 has yielded theoretical relative bandwidths of 40–50 percent using the FA [3], and experimental bandwidths of nearly 30 percent in X-band waveguide have been measured [5]. However, Fig. 3 shows that the FA can indicate a performance that is too optimistic. The SA shows a performance that is somewhat inferior to the FA, but which agrees more closely with the measured characteristics. It is particularly satisfying that reflection loss agreed well with the measurements, because 3-port circulator design is frequently based upon inspection of the reflection coefficient (or input impedance) characteristic as a function of frequency.

In order to test the 4-port theory, but not necessarily to make a circulator, two of the 3-port composite posts were tested in a 4-port waveguide junction. In Fig. 4 the FA and SA are shown with the

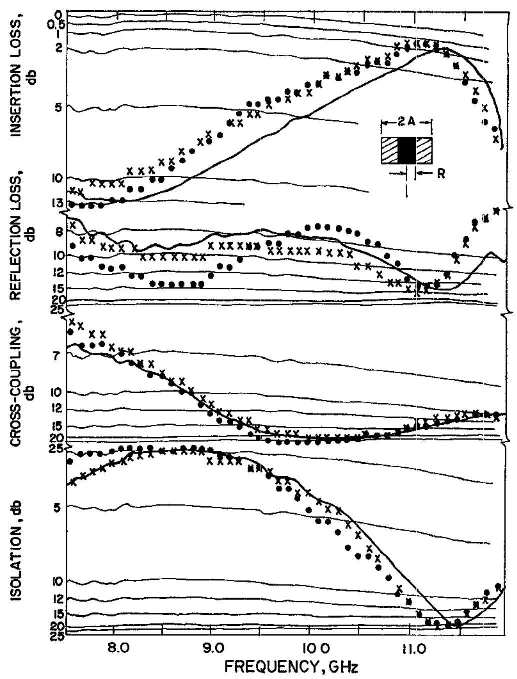

Fig. 4. 4-port performance of a TT1-390 ferrite rod, $R=0.350$ cm. (———, measured; · · ·, first approximation; × × ×, second approximation.)

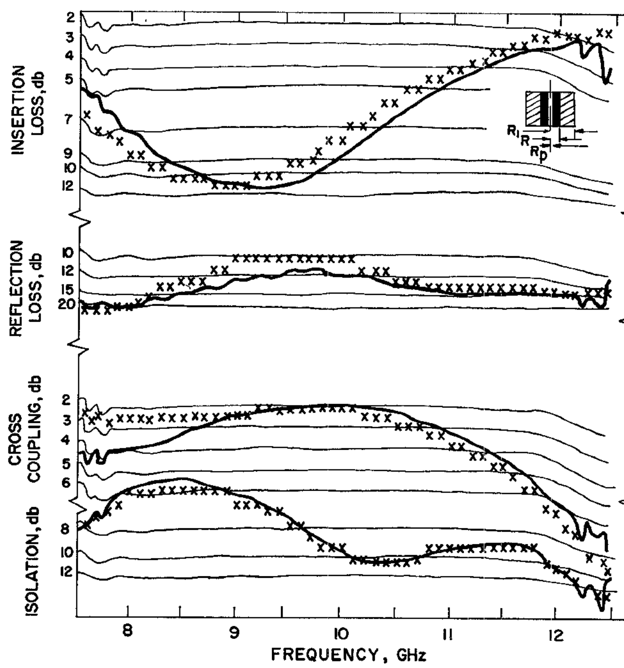

Fig. 5. 4-port performance of a TT1-390 ferrite rod, metal pin, and dielectric sleeve. $R_1=0.775$ cm, $R=0.275$, $\epsilon_{r1}=2.08$. (———, measured; × × ×, second approximation.)

measured results. There is relatively little difference between the two sets of computed curves, except for the reflection loss, and the agreement between the second computations and experiment is good. It should be noted that the insertion loss is minimum, while reflection loss and isolation (ports 1–4) are maximum at 11.5 GHz. But the cross-coupling (ports 1–3) curve reaches a minimum in the range 9.5–11.0 GHz, and has risen considerably at 11.5 GHz. This is the type of behavior which restricts the bandwidth of these devices. As another example, the thin-pin configuration provides good agreement with the SA theory in Fig. 5, and it appears that circulator action might be occurring about 12.5 or 13 GHz, but there is no indication of other than narrow-band performance.

Finally, two unsuccessful results should be mentioned briefly. Some results of empirical 4-port circulator design using full-height structures are available in the literature [6], [7], but attempts to predict these results were unsuccessful. This is attributed to a lack of reliable ferrite data, and the sensitivity of 4-port performance to small changes in material properties and dimensions. Also, an unsuccessful attempt was made to obtain a good (predicted) 4-port circulator using the design procedure suggested by Helszajn and Buffler [8]. Further investigations are under way.

IV. Discussion and Conclusions

Davies' theory of the symmetrical m-port nonreciprocal waveguide junction has been successfully extended to a higher approximation for $m=3$ and 4. The SA agrees very well and resolves some of the previously unexplained discrepancies displayed by the FA. The SA provides some of the "fine structure" and provides a more realistic estimate of bandwidth and loss between ports. This is particularly useful for the potentially broad-band thin-pin 3-port structures.

The penalty we have paid for improved calculations is an increase in computer time necessary to make them. The increase in the number of terms in n requires many more calculations and, especially for the more complex structures, the increase in computation time is marked. As examples let us consider a simple junction, the 3-port with a ferrite post, and then the more complicated 4-port with a ferrite post, metal pin, and dielectric sleeve. With the 3-port, the FA and SA take approximately 0.55 and 1.65 min, respectively, for a set of computations over the waveguide bandwidth. With the 4-port, the times are approximately 1.63 and 4.22 min, respectively. These times include approximately 0.180 min for the graphical printout in the form of a simulated swept-frequency plot. The programs might be made more efficient, and the machine was a Burroughs B-5500.

It should be emphasized that we now have a proven analysis technique for a particular class of waveguide structures; we do not have a circulator design program. Any successful design that is developed using this program is still likely to be the result of experience just as it would be in the laboratory, but the difference is that it would have been found in less time or it would be the best of a larger number of attempts. However, one would still not have any information as to whether this was the "best" design in any sense of the word.

Two avenues may now be followed. A performance figure of merit could be defined and optimization routines developed that would automatically maximize the figure of merit. This is probably the most straightforward extension, and one that will fill a real need. It will, however, be complicated by the many parameters present in all potentially attractive structures, the complexity of the behavior with parameter changes (particularly in the 4-port), and the fact that a global maximum is required.

The authors feel that while analysis and optimization are the only good design procedures within reach, the real requirement is for a successful circulator synthesis procedure.

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Uniform Multimode Transmission Lines

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***Abstract*—A consistent normal-mode theory for coupled uniform transmission lines is developed which properly accounts for the fact that some propagation constants may be equal. Explicit responses are calculated. The results are interpreted in terms of transmission-line networks which consist of simple lines and which are coupled via conductor sharing.**

The design and understanding of high-speed computer circuitry require the transient solution of the n-wire uniform transmission-line problem. The interconnections are normally treated as coupled lossless lines with a single propagation constant. If, however, one (or more) of the system members is loaded periodically, the model fails since several propagation constants may occur. Thus there is need for an analysis that allows for partial eigenvalue degeneracy and yields the transient response.

It is assumed that the n-wire and ground system is described by [1]

$$\left.\frac{dV}{dx}\right] = -[Z]I] \tag{1a}$$

$$\left.\frac{d}{dx} I\right] = -[Y]V] \tag{1b}$$

$$\left.\frac{d^2}{dx^2} V\right] = [Z][Y]V]. \tag{1c}$$

Here the conductor-to-ground voltages and the conductor currents are Laplace-transformed quantities. The matrices $[Z]$ and $[Y]$ are x independent. The propagation constants are the solutions of (2) [2]:

$$\left| [Z][Y] - \gamma^2[\delta_{ij}] \right| = 0. \tag{2}$$

To form the modal matrix, it is assumed initially that the n eigenvalues of $[\Gamma]=[Z][Y]$ are distinct. Then the solution for the kth mode for a system that extends from $x=0$ to ∞ becomes

$$V_k] = [\epsilon^{-\gamma_k x}]V_k(0)] \tag{3}$$

where the propagation matrix is diagonal. The total voltage at $x=0$, the sending end, becomes

$$\left.\sum_{k=1}^{n} V_k(0)\right] = [\alpha][V_{ii}(0)]1] \tag{4}$$

where $[V_{ii}(0)]$ is a diagonal matrix with elements $V_{kk}(0)$, i.e., the voltage from conductor k to ground for the kth mode. This form is possible if the modal matrix is constructed as follows. Substitution of γ_k into (1c) and suppression of the kth row of Γ yields

$$\begin{aligned} -\Gamma_{ik}]V_{kk}] &= [A]V_{ik}], && i \neq k \\ A_{ii} &= \Gamma_{ii} - \gamma_k^2, && i \neq k \\ A_{ij} &= \Gamma_{ij}, && i \neq j,\ i \neq k,\ j \neq k. \end{aligned} \tag{5}$$

This equation may be solved for the eigenvector $V_k]$. In particular,

$$\alpha_{i,k} \equiv \frac{V_{ik}}{V_{kk}} \tag{6}$$

defines the members of the modal matrix $[\alpha]$. The voltages on the infinite system become

$$V] = [\alpha][V_{ii}][\epsilon^{-\gamma x}]1] \qquad [\epsilon^{-\gamma x}]_{ij} = \epsilon^{-\gamma_i x}\delta_{ij}. \tag{7}$$

The corresponding total current is readily shown to be

$$I] = [Y][\alpha][\gamma]^{-1}[\epsilon^{-\gamma x}][V_{ii}]1]. \tag{8}$$

The assumption that the system is excited at $x=0$ by $I_S]$ and ter-

Manuscript received March 15, 1971; revised December 3, 1971.
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